MEASUREMENT OF FERROMAGNETIC PERMEABILITY AT MICROWAVE FREQUENCIES*

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ABSTRACT. The relative apparent permeabilities of soft iron and nickel (in the form of thick plates) are measured at a free-space wavelength of 3.2 cms. The method of measurement, which employs a resonant cylindrical cavity, is described. Expressions are derived relating the permeability, μ_R with the cavity parameters and the resonant frequency. Approxmate relations, from which μ_R can be easily calculated, are given. The method gives value of μ_R accurate within a few per cent. The effect of machining inaccuracy of one of the end walls of the cavity on the measurement of R. F. permeability is also discussed.

1 INTRODUCTION

The magnetic properties of ferromagnetic materials at v. h. f. and higher frequencies have been the subject of extensive studies in recent years. With the development of microwave technique during the war, it has been possible to extend these investigations to the frequency ranges of these waves. It has been found that the behaviour of ferromagnetic materials at such frequencies is not at all simple. Amongst other interesting effects, mention may be made of the dependence of the permeability on the frequency, on the magnitude and direction of the polarising d. c. magnetic field and on the surface conditions, temperature and tension of the sample.

The R. F. relative permeability can be expressed in terms of two real numbers, μ_R and μ_L, called apparent permeabilities, or by a complex number. It has been shown that $\mu_R > \mu_L$ and that with the increase of frequency both μ_R and μ_L decrease. The different methods that have been developed for the determination of R. F. permeability are all based on on the measurement of the impedance of a circuit element containing the ferromagnetic material under test. Some examples of such methods, as have been employed, are given below.

Johnson and Rado (1949) have determined both components of the complex permeability in the frequency range 200 to 975 Mc/s (as a function of a polarising magnetic field parallel to the H. F. field) by measuring the changes in the quality factor and in the resonant frequency of a half wave coaxial resonator, the inner conductor of which was of the ferromagnetic metal under test. Briks (1948) has determined the complex permeability by measuring the relative characteristic impedance and propagation

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constant of a section of a rectangular guide filled with the substance in question in paraffin wax base. Griffiths (1946) has used a cylindrical cavity, with one of its end walls covered with a thin layer of the metal under test (electrolytically deposited), and has investigated the ferromagnetic resonance of the substances by measuring the Q of the cavity.

In the method adopted for the present investigation, a cylindrical cavity made of a non-magnetic material with one detachable end plate is used. The resonant frequency and the quality factor of the cavity are measured, once when the detachable end plate is made of the non-magnetic material and again when it is made of the ferromagnetic material under test. These measurements enable one to evaluate the μ_R of the test material. The method is more accurate and, from certain expressions that have been derived, the calculations are made very simple. The method, is, however, not suitable for measurement of μ_L, as this involves changes in the resonant frequencies of the cavity, too minute (0.0001%) to be detected with the equipment. Experiments are, however, in progress to develop a sufficiently precise method and the results will be communicated in due course.

2. EXPERIMENTAL ARRANGEMENT AND METHOD OF MEASUREMENT

A schematic diagram of the experimental arrangement is shown in figure 1 (*a*) and a photographic view of the same is given in figure 1 (*b*). Brief

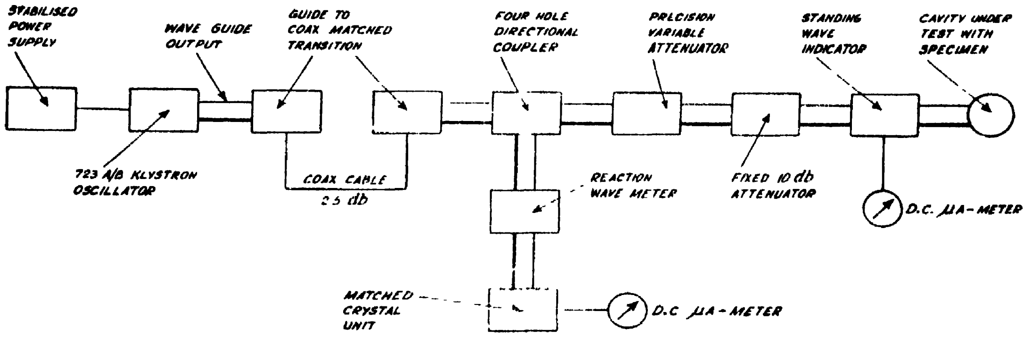

FIG. 1(*a*)

Block diagram of the experimental arrangement.

descriptions of the main units of the arrangement are given below. A 723 A/B klystron c. w. oscillator, having a stabilised power supply is used as the signal generator. A smooth variation of the frequency is obtainable by controlling the reflector voltage of the klystron round the optimum value. The singal generator feeds, *via* a coaxial-to-waveguide transition unit, into a four-hole directional coupler; the main branch of the directional coupler is utilised for the standing wave measurements and the subsidiary branch of the same for frequency measurements and monitoring. A fixed 10 db. attenuator and a variable precision

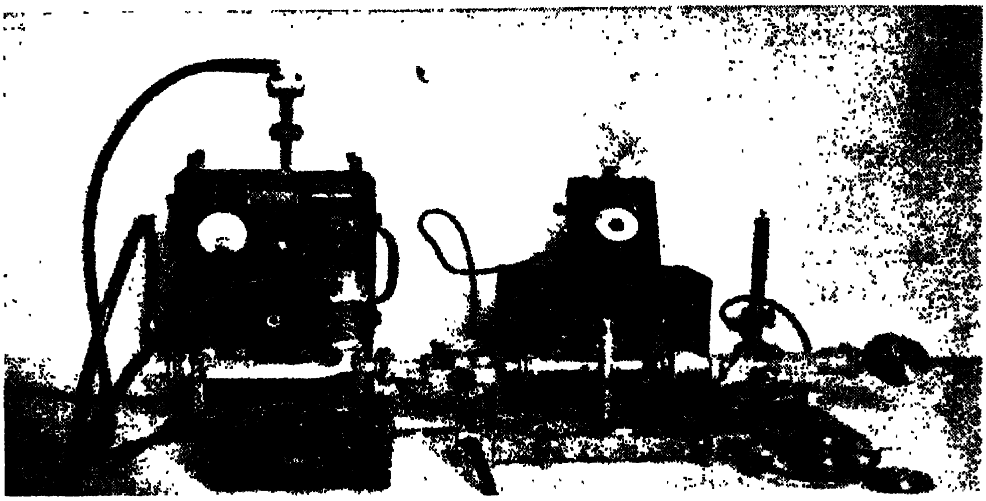

FIG. 1(b)
Photographic view of the experimental arrangement

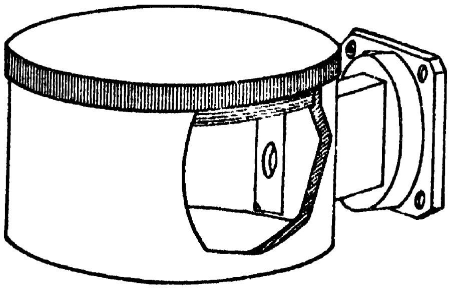

FIG. 2
Test cavity resonator with detachable end wall.

attenuator are interposed between the directional coupler and the standing wave measurement line (waveguide). The minimum attenuation between the singal generator and the load (i. e. the test cavity) is thus, including the 2.5 db. loss in the co-axial cable, 12.5 db. which is sufficient for eliminating any interaction between the two during the standing wave measurements. This means that the generator is always looking into a matched load.

The standing wave measurements are carried out by a precision wave-guide slotted section together with a broad band probe detector unit. The probe detector unit is tunable over a frequency range of 1,000 to 12,400 Mc/s and consists of three co-axial cylinders acting as two co-axial line pairs in series. The depth of the probe tip insertion inside the guide is adjustable and is always kept less than 1/32″ inch to minimise the distortion of the field inside the guide. The probe supplies power to a IN 23 crystal, which is held in pressure contact with the innermost conductor (the same conductor as the probe) of the co-axial line system and is used for R. F. detection. A 40 μA full scale unipivot type d. c. microammeter measures the detected current.

The cavity under test is a right circular cylinder of brass with a detachable end wall, either of brass or of the ferromagnetic metal, as shown in figure 2. The cavity is coupled to the waveguide section (I. D. 0.9″ × 0.4″), the broad side of which is parallel to the cavity axis, by a small circular hole (0.285″ dia.) in the middle of the cylindrical wall of the cavity. By this method of coupling, only the TE_{011} mode is excited. Further, the end plates of the cavity are made accurately perpendicular to the cavity axis to prevent the simultaneous excitation of the degenerate TM_{111} mode which is undesirable. The resonant frequency of the cavity is dependent, amongst other factors, on μ of the material of the end plate.

The voltage standing wave ratio (V. S. W. R.) at any frequency due to the test cavity was measured as follows. The detector carriage was placed at the minimum voltage position and the detected current and the position of minimum noted. The probe was then moved to the adjacent maximum position and the detected current was brought back to the previous minimum current value by the precision variable attenuator. The attenuation introduced by this attenuator was thus a measure of the V. S. W. R. The readings were checked for several minima and maxima.

The coupling hole between the test cavity and the waveguide section acts as a lossless transformer At a certain reference plane in the cavity coupling system, the waveguide line (characteristic impedance Z_0 in the equivalent transmission line circuit) looks, at resonance, into a pure resistance, R which accounts for the total loss inside the cavity. In general, the ratio Z_0/R may be greater, equal to or less than unity depending, amongst other parameters, on the diameter of the hole and the wall thickness. Referring to the Smith chart, it can be seen that if $Z_0/R > 1$, the position of the minimum in the slotted standing wave measuring guide shifts from that at resonance by a quarter guide wavelength at frequencies far off resonance; whereas, a shift of the minimum, much less than a quarter guide wavelength, indicates that Z_0/R is less than unity. By applying the above test the coupling parameter, Z_0/R of the test cavity was, in all cases, found to be less than unity.

For the accurate measurement of frequency difference off resonance, a wave meter is used in the secondary branch of the directional coupler. The wave meter (figure 3) is of a special reaction type and consists essentially of a brass cylindrical cavity. For the adjustment of its centre frequency, a brass rod 9/32 in. diameter is fitted and soldered (after adjustment) in the centre of one of the faces along the axis of the cavity. For the fine adjustment of frequency a much thinner rod, 1/8 in. in diameter, is fitted to a micrometer screw head to slide parallel to the axis at a distance of 23/32 in, from the centre. The wavemeter is calibrated separately and the wavemeter guide circuit is terminated in a matched IN21 crystal unit. The crystal current also served the purpose of monitoring the R.F. power when the wavemeter was tuned off resonance. It is to be noted that since the

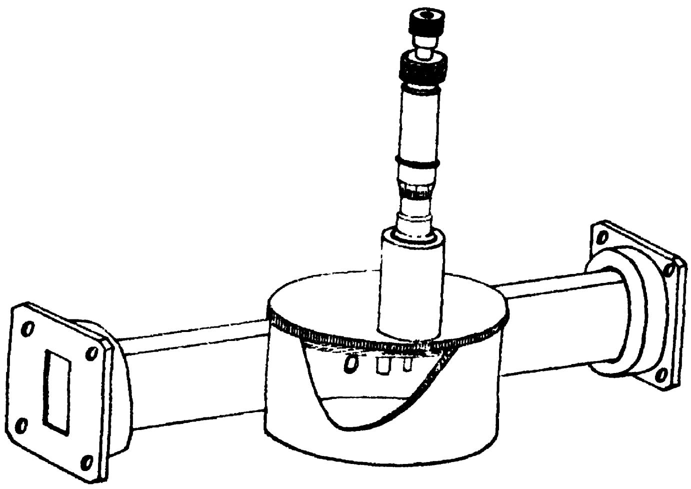

FIG. 3
Cutway view of the reaction type cavity wavemeter showing the fixed and the movable rod inside the cavity.

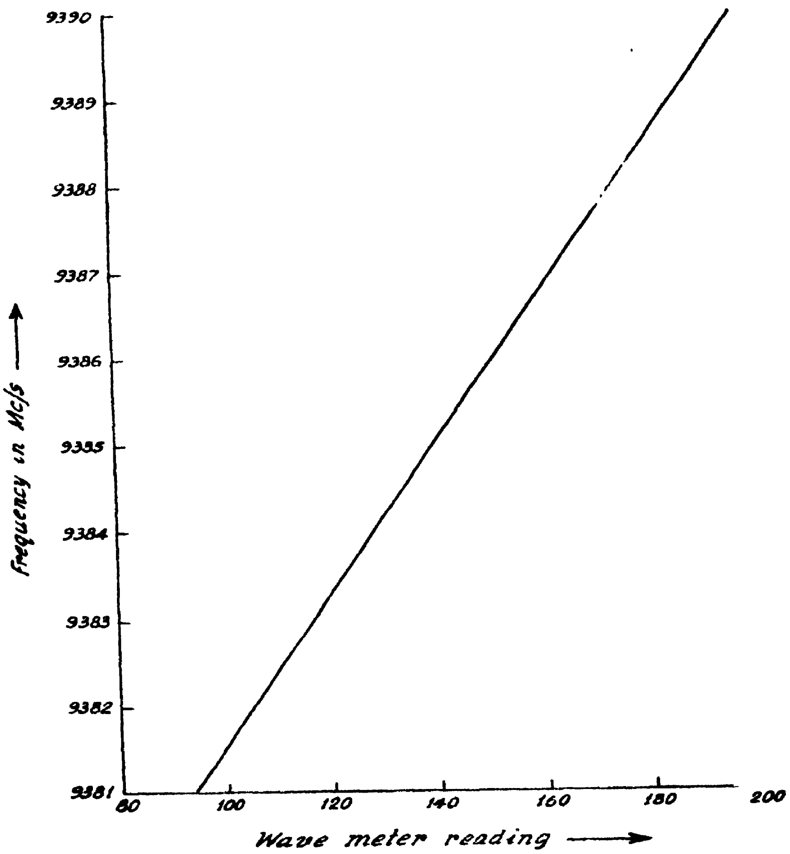

FIG. 4
Calibration curve of the reaction type cavity wavemeter.

attenuation introduced by the directional coupler between the primary and the secondary guide circuit is 20 db, it is ensured that there is no pulling effect between the singal generator and the wavemeter circuit. As the quality factor of the test cavity is very high, the difference in frequency that is to be determined round the resonant frequency of the test cavity, is necessarily very small. The reaction type of cavity wavemeter is, therefore, constructed having a direct reading accuracy of approximately 0.04 Mc/s and tunable over a frequency range of 9380 to 9398 Mc/s. In the small frequency range used in this experiment, the frequency varies linearly with the wavemeter setting as shown in figure 4. A sharp dip in the crystal current indicates the frequency. The constancy of this minimum reading is a check that the frequency of the signal generator remains steady during a particular measurement.

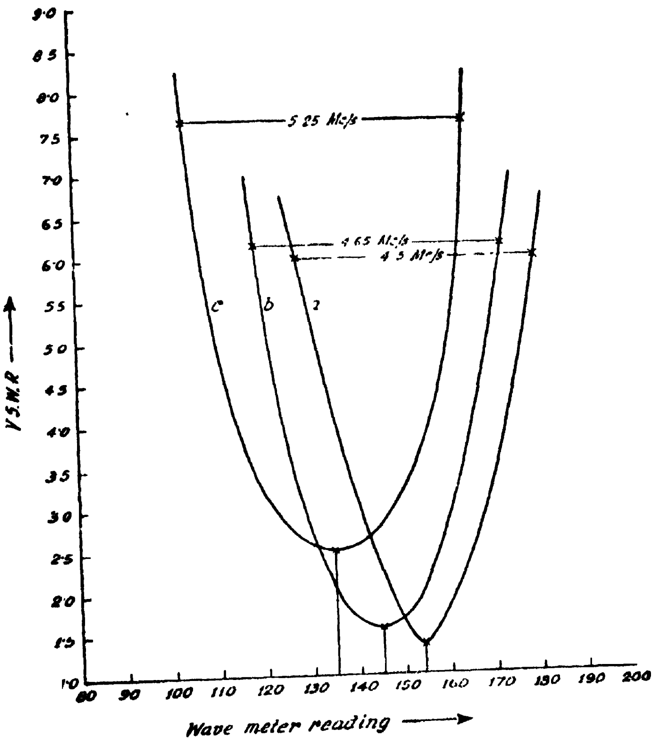

FIG. 5

V. S. W. R. in the waveguide versus frequency due to the test cavity resonator with various end plates. Curve *a* refers to the case of brass end plate; curves *b* and *c* are for nickel and soft iron end plates respectively.

V. S. W. R. *versus* frequency measurements were performed with the test cavity having detachable end plates made of brass, nickel, and soft iron. The results obtained are shown in figure 5.

In order to check the constancy of the geometry of the cavity, V.S. W.R. *versus* frequency measurements were carried out with several end plates of the same material. The measurements showed that though the quality factors were the same, the maximum deviation of resonant frequency for the different end plates of same material due to machining inaccuracy was ±0.4 Mc/s. This amount of uncertainty of the resonant frequency, though small, is large enough to mask the minute change in the resonant frequency that would be produced due to change in the value of μ_L of the ferromagnetic substance.

3. THEORETICL ANALYSIS

The cylindrical cavity resonator used in this experiment has one end wall made either of brass or of some ferromagnetic metal. Ferromagnetic metals possess complex permeability and, as shown hereafter, the performances of a cavity (*e.g.* the resonant frequency and the quality factor) with an end plate made of such a material is different from that of a cavity made entirely of a non-magnetic material, *e.g.* brass. In what follows we shall derive expressions relating the R. F. permeability with the resonant frequency and quality factor of a cylindrical cavity, having one end plate made of some ferromagentic material.

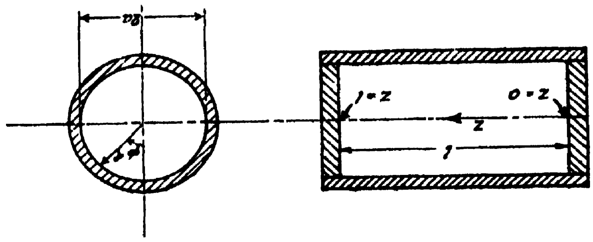

FIG. 6

Cylindrical cavity resonator illustrating the coordinate axes.

A lossless cylindrical air-dielectric cavity, such as shown in figure 6, when excited in the TE_{011} mode, has an oscillating electro-magnetic field given by (Sarbacher and Edson, 1943),

$$H_z = -jJ_0\left(\frac{p}{a}r\right)\sin\left(\frac{\pi z}{l}\right)e^{j\omega t}$$

$$H_r = j\beta\left(\frac{a}{p}\right)J_1\left(\frac{p}{a}r\right)\cos\left(\frac{\pi z}{l}\right)e^{j\omega t}$$

$$E_\varphi = -\omega\mu_0\left(\frac{a}{p}\right)J_1\left(\frac{p}{a}r\right)\sin\left(\frac{\pi z}{l}\right)e^{j\omega t} \quad \ldots \quad (1)$$

where, J_0 and J_1 are Bessel Functions of the first kind, of zero and first order respectively ;

$$\beta=\left|\left(\frac{\omega}{c}\right)^2-\left(\frac{p}{a}\right)^2\right|^{1/2}=\frac{\pi}{l} \quad \ldots \quad (2)$$

is the phase constant of the corresponding travelling wave mode ; $p=3.832$, the first root of the equation, $J_0'(x)=0$,

$$c=\frac{1}{\sqrt{\mu_0\epsilon_0}}=3\times 10^8 \text{ metres/sec.}$$

is the velocity of light in free space.

Equations (1) are also applicable, without serious error, to cylinders with high Q, *i.e.* low loss.

The maximum electric energy, W_E stored inside the cavity is

$$W_{EO}=\tfrac{1}{2}\epsilon_0\int_0^{2\pi}\int_0^a\int_0^l |E_\phi|^2\, r dr d\phi dz$$

Or, substituting for E_ϕ from equation (1),

$$W_{EO}=\tfrac{1}{4}\left(\frac{a}{p}\right)^2 V\mu_0\left(\frac{\omega_0}{c}\right)^2 J_0^2(p), \quad \ldots \quad (3)$$

where, $V=\pi a^2 l$ is the volume of the cylindrical cavity.

The quality factor, Q of the cavity, without external loading, is given by

$$Q_0=\frac{\omega_0 W_{E0}}{2P_{z0}+P_{r0}} \quad \ldots \quad (4)$$

where $\qquad \frac{\omega_0}{2\pi}=f_0=$ the resonant frequency

$P_{r0}=$ power dissipated in the cylindrical wall perpendicular to the r-axis, and $P_{z0}=$ power dissipated in each of the two faces perpendicular to the z-axis.

The power dissipated in the cylindrical wall is

$$P_{r0}=\tfrac{1}{2}Re[Z_t]\int_0^l\int_0^{2\pi} |H_{tan}|^2\, a d\phi dz, \quad \ldots \quad (5)$$

where, $|H_{tan}|$ is the amplitude of the tangential component of the magnetic field at $r=a$

and $$Z_t=\sqrt{\frac{j\omega_0\mu_0}{\sigma_1}} \quad \ldots \quad (6)$$

is the intrinsic impedance (*i.e.* the ratio of transverse electric to magnetic field) of the wall material, and $\sigma_1=$ electrical conductivity of the wall material.

Substitution of (6) in (5) yields

$$P_{r0}=\tfrac{1}{4}\sqrt{\frac{\pi f_0\mu_0}{\sigma_1}}A_r J_0^2(p), \qquad (7)$$

where, $A_r=2\pi al=$ the cylindrical surface area.

The power dissipated in each end wall is

$$P_{e0}=\tfrac{1}{2}Re\,[z_t]\int^{2\pi}\int^{a}|H_r|^2\,rd\phi dr, \qquad (8)$$

where, $|H_r|$ is the amplitude of the tangential component of the magnetic field at the end walls, *i.e.* at $z=0$ or $z=l$.

Then substituting for H_r from equation (1) and for Z_t from equation (6), we have for the power dissipated in one end wall,

$$P_{e0}=\tfrac{1}{2}\sqrt{\frac{\pi f_0\mu_0}{\sigma_1}}\beta^2\left(\frac{a}{p}\right)^2 A_e J_0^2(p), \qquad (9)$$

where, $A_e=\pi a^2$, the area of one end wall.

In the above, the wall material is assumed to be made of a non-magnetic material (*e.g.*, brass). If, however, the wall material be of a ferromagnetic metal (*e.g.*, nickel), then μ is complex and is given by

$$\mu=\mu_1-j\mu_2=|\mu|e^{-j\psi}. \qquad \ldots \quad (10)$$

where $\quad |\mu|=\sqrt{\mu_1^2+\mu_2^2}$ and $\psi=-\tan^{-1}\frac{\mu_2}{\mu_1}$.

The apparent permeabilities, μ_R and μ_L are related to μ as follows :

$$\mu_R=(|\mu|+\mu_2)/\mu_0 \text{ and } \mu_L=(|\mu|-\mu_2)/\mu_0.$$

For such case we have, therefore,

$$Z_t=\sqrt{\frac{j\omega\mu}{\sigma_2}}$$

$$=\sqrt{\frac{\omega}{2\sigma_2}}\left[\sqrt{|\mu|+\mu_2}+j\sqrt{|\mu|-\mu_2}\right], \qquad (11)$$

where, σ_2 is the conductivity of the ferromagnetic metal.

Hence, if a cylindrical cavity has one of its end walls made of a ferromagnetic material, the power dissipated in this wall will be given by

$$P_e=\frac{1}{2}\sqrt{\pi f(|\mu|+\mu_2)/\sigma_2}\,\beta^2\left(\frac{a}{p}\right)^2 A_e J_0^2(p) \qquad (12)$$

(Note: It is assumed in the above equations that the field configuration inside the cavity is not materially disturbed due to the presence of the ferromagnetic metal. This assumption is justified as the quality factor of the cavity remains high even with the ferromagnetic end-wall).

Now, when one of the end walls of the cavity is made of a ferromagnetic metal and the rest is of non-magnetic material, the quality factor is given by

$$Q_f=\frac{\omega_0 W'_{EO}}{P'_{r0}+P_e+P_{e0}} \qquad \ldots \quad (13)$$

where, $\frac{\omega}{2\pi} = f$, is the new resonant frequency of the cavity and the dashed symbols stand for the respective quantities at a frequency

It, therefore, follows from equations (4) and (13), that

$$\frac{1}{Q_f} - \frac{1}{Q_0} = \frac{P'_{r0} + P_Z + P'_{t0}}{\omega W_{E_0}'} - \frac{P_{r0} + 2P_{t0}}{\omega_0 W_{E_0}} \quad \ldots \quad (14)$$

Since the conductivity is very high, both for the non-magnetic and for the ferromagnetic metal, we can put $\beta = \frac{\pi}{l}$ in the equations (9) and (12). Substitution for P_r and P_t from appropriate equations, with due regard to the frequency, in equation (14) then yields,

$$\frac{1}{Q_f} - \frac{1}{Q_o} = \frac{1}{4\pi^3}\sqrt{\frac{\pi}{\mu_0\sigma_1}}\,\frac{p^2}{a^3}\,c^2\left[f^{-5/2} - f_0^{-5/2}\right]$$
$$+ \frac{1}{4\pi}\sqrt{\frac{\pi}{\mu_0\sigma_1}}\,\frac{c^2}{l^3}\left[f^{-5/2} - f_0^{-5/2}\right]$$
$$+\left[\frac{1}{4\pi}\sqrt{\frac{\pi\mu_R}{\mu_0\sigma_2}}\,\frac{c^2}{l^3}\,f^{-5/2} - \frac{1}{4\pi}\sqrt{\frac{\pi}{\mu_0\sigma_1}}\,\frac{c^2}{l^3}\,f_0^{-5/2}\right] \quad \ldots \quad (15)$$

Putting $f_0 = f(1+\delta)$, where $\delta << 1$ and remembering that $\sigma_1/\sigma_2 \approx 1$, simplification is usually possible by considering the relative values of the different terms. Each of the first two terms is of the order of 10^{-3}. If then $\mu_R = 1 + q$ say, the third term is about q/δ times the other terms. Thus,

$$\frac{1}{Q_f} - \frac{1}{Q_o} = \frac{1}{4\pi}\sqrt{\frac{\pi}{\mu_0\sigma_1}}\,\frac{c^2}{l^3}\,f_0^{-5/2}\left[\left(\sqrt{\frac{\sigma_2}{\sigma_2}}\right)\mu_R^{1/2} - 1\right] \quad \ldots \quad (16)$$

Q_o and Q_f may be obtained from the standing wave measurement data as follows:

When a cavity coupling system has a single emergent transmission line, the V.S.W.R., produced in the line is given (Montgomery, 1947) by (see figure 7),

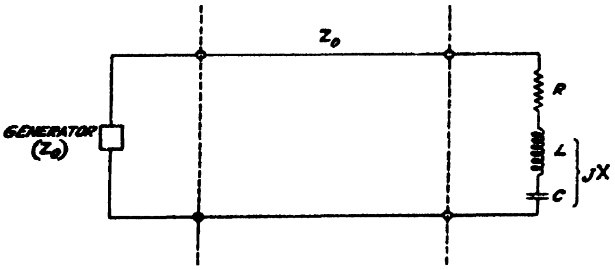

FIG. 7

Equivalent circuit of the cavity coupling system, at a particular reference plane, terminating a waveguide transmission line

$$\text{V.S.W.R.} = \frac{\sqrt{(Z_0+R)^2+X^2}\,| + |\sqrt{(Z_0-R)^2+X^2}}{\sqrt{(Z_0+R)^2+X^2}\,| - |\sqrt{(Z_0-R)^2+X^2}|} \quad (17)$$

where, $R+jX=R+j2RQ\frac{\Delta f}{f_0}$ is the equivalent line terminating impedance at some appropriate reference plane of the cavity coupling system, Z_0 is the characteristic line impedance in the equivalent transmission line circuit, Q is the unloaded quality factor of the cavity, and f_0 is the resonant frequency of the cavity.

At resonance, *i.e.*, $\Delta f=0$, the V.R.W.R from equation (17) is

$$(i)\ \rho=R/Z_0, \text{ if } R>Z_0,$$

or,

$$(ii)\ \rho=Z_0/R, \text{ if } R<Z_0.$$

Since the cavity coupling in this case is such that $R/Z_0>1$,

$$\rho=R/Z_0, \quad \ldots \quad (18)$$

For the frequencies, $f_0\pm\Delta F$ on either side of resonance, $X=\pm(Z_0+R)$ and the V.S.W.R. from equations (17) and (18) becomes

$$\rho_1=\frac{1+\rho+\sqrt{1+\rho^2}}{1+\rho-\sqrt{1+\rho^2}} \quad \ldots \quad (19)$$

From the resonance curve (figure 5) the value of $2\Delta F$ is obtained by knowing ρ_1.

The unloaded quality factor of the cavity thus is

$$Q=\frac{f_0}{2\Delta F}\left(1+\frac{1}{\rho}\right). \quad \ldots \quad (20)$$

4. EXPERIMENTAL RESULTS AND DISCUSSION

Test cavity dimensions:

Diameter, $2a=4.95$ cms.

Length, l $=2.60$ cms,

Coupling hole diameter $=0.285''$

The experimental results obtained are summarised in Table I.

TABLE I

Material	Resonant frequency f_0 in Mc/s.	V.S.W.R at resonance, ρ	ρ_1	$2\Delta F$ in Mc/s.	Conductivity in mhos/metre $\times 10^{-7}$	Q	μ_R
Brass	9386.2	1.37	6.03	4.50	1.57	3620	1
Nickel	9385.45	1.55	6.17	4.65	1.3	3320	4.75
Soft iron	9384.55	2.5	7.65	5.25	1.0	2500	36

The experimentally obtained values of μ_R are given in the last column of the table. These values, namely 36 for soft iron and 4.75 for nickel, agree closely with the values as obtained by other workers.

It is to be noted that the slight differences in the resonant frequencies, as given in the second column of the table for the cavity of same dimensions but of different materials for one of the end wells are to be ascribed more to machining inaccuracy than to differences in the value of μ_L of the materials used. This was checked by an independent measurement in which the resonant frequencies of the cavity, with different end plates of the same material, were measured. The results obtained are given in Table II.

TABLE II.

End plate sample number	Resonant frequency in Mc/s.
1	9386 1
2	9385.8
3	9385.6
4	9386.4
5	9386.3
6	9386 2

Since the maximum error in the measurement of resonant frequency due to changes in geometry of the cavity is ± 0.4 Mc/s in 9385 Mc/s, *i.e.*, 0.004%, it is seen from equation (21) that the error in Q-measurements for this reason is also of the same order. This will produce negligible error in the value of μ_R.

5. ACKNOWLEDGMENT

The authors take this opportunity to thank Professor S. K. Mitra for his encouragement and critical discussion and for his help in the preparation of the paper.

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CORRIGENDUM

ON THE SOLUTIONS OF MAXWELL'S EQUATIONS IN AN INFINITE MEDIUM, Etc.

By K. V. KRISHNA PRASAD

The material presented in my two papers entitled "On the approximate solutions of Maxwell's equations in an infinite medium with regions of finite conductivity" and "Rigorous solution for the case of electromagnetic wave propagation along a circular wave guide of finite conductivity", published in the Indian Journal of Physics, of August and September, 1951, respectively, are very much based on the material presented in the thesis by Dr. Glenn M. Roe, of March 1947, available from the University of Minnesota Library U. S. A. Several of the equations presented in the above papers are wrong and the correct equations are found in the thesis referred to above.